Calculus II Spring 2010 Worksheet 4

Your Name: **Instructor: Steven Clontz**

Draw a box around your final answer. You must show all work to receive credit.

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1. Power Series (8.7)

   (a: #4) Find the interval and radius of convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n}$.

   (b: #14) Find the interval and radius of convergence for $\displaystyle\sum_{n=0}^{\infty} \frac{(2x+3)^{2n+1}}{n!}$.

(c: Based on #44)

The exponential function e^x may be represented by the power series (with interval of convergence $-\infty < x < \infty$)

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \dots$$

Using term-by-term differentiation and integration, show that e^x has a derivative of e^x and an integral of $e^x + C$.

(See 8.7 #1-38 and #41 for more examples of questions about Power Series.)

2. Taylor and Maclaurin Series (8.8)

(a: #2) Find the Taylor Polynomial $P_3(x)$ of order 3 generated by $f(x) = \dfrac{1}{1+x}$ at $x = a$.

(b: #14) Find the Maclaurin Series generated by $\sin\left(\dfrac{x}{2}\right)$.

(c:) Using the geometric series $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ with interval of convergence $-1 < x < 1$, find a power series to represent $f(x) = \frac{2}{3(1-x)^2}$, give its interval of convergence, and write out the first four terms of the series.

(d:) Now find the Taylor Polynomial $P_4(x)$ of order 4 generated by $f(x) = \frac{2}{3(1-x)^2}$ at $x = 0$.

3. Convergence of Taylor Series (8.9)

(a:) Show that $\cos(x)$ converges to its Taylor Series $\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k}}{(2k)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$ for any real number x.

(b:) Express $\cos(x^2)$ as a power series.

(c:) Express $\int \cos(x^2)dx$ as a power series.

(d:) Evaluate $e^{1+3\pi i}$.

(See Section 8.9 #1-18 for more examples of Convergence of Taylor Series examples. Practice finding $\lim_{n\to\infty} R_n$ in particular.)

4. Binomial Series (8.10)

(a:) Find the first four terms of the expansion of $(x+1)^{12}$.

(b: #6) Find the first four terms of the binomial series for $\left(1-\frac{x}{2}\right)^{-2}$.

(See 8.10 #1-14 for more examples concerning binomial series.)

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5. Polar Coordinates (9.1)

(a:) Graph the polar coordinate $(-3\sqrt{2}, \frac{\pi}{4})$ and give three other equivalent polar coordinates for that point.

(b:) Give the Cartesian, that is, (x, y) coordinates for the polar coordinate $(-3\sqrt{2}, \frac{\pi}{4})$.

(c: #10) Graph the polar inequality $1 \leq r < 2$.

(d: #38) Graph the polar equation $r \sin\theta = \ln r + \ln\cos\theta$ by first changing it into a relation of x and y.

(See 9.1 #1-62 for more examples of using polar coordinates.)

6. Graphing in Polar Coordinates (9.2)

(a:) Find the slope of the line tangent to the polar graph of $\sqrt{2} + r \sec\theta = 0$ at the polar coordinate $(-1, \pi/4)$.

(b:) Show that the polar graph of $r + \sin(2\theta) = 2$ is rotationally symmetric about the origin.

(c:) Show that the polar graph of $r = 2 - 2\cos\theta$ is symmetric across the x-axis.

(d:) Sketch the polar graph of $r = 2 - 2\cos\theta$.

(See 9.2 #1-16 for more examples of graphing with polar curves.)

7. Areas and Lengths in Polar Coordinates (9.3)

(a:) Find the length of the perimeter of the cardoid given by $r = 2 - 2\cos\theta$.

(b:) Find the area inside of the cardoid given by $r = 2 - 2\cos\theta$.

(See 9.3 #1-6, #17-25 for more examples of polar areas and curves.)