Calculus II Spring 2010 Worksheet 1

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Draw a box around your final answer. You must show all work to receive credit.

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1. Volumes by Slicing (6.1)

   (6.1 #2) A solid is made up of of circular cross-sections running from $x = -1$ to $x = 1$, centered on and perpendicular to the line $y = 1$. The diameters of these cross sections on the xy plane run from the the parabola $y = x^2$ to the parabola $y = 2 - x^2$. Find its volume.

   (6.1 #8) The base of a solid is the disk $x^2 + y^2 \leq 1$. Cross sections perpendicular to the y-axis are isosceles right triangles with one leg on the disk. Find the volume of this solid. (Hint: The width of a triangular cross-section is given by $2\sqrt{1 - y^2}$.)

   (See 6.1 #1-10 for more examples.)

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2. Volume of Solids of Rotation: Disk Method (6.1)

   (6.1 #18) Use the disk method to find the volume of the solid obtained by rotating the region bounded by the curves $y = x^3$, $y = 0$, and $x = 2$ around the x-axis.

   Use the disk method to find the volume of the solid obtained by rotating a triangle with vertices at (1,5), (3,4), and (1,1) around the line $x = 1$.

3. Volume of Solids of Rotation: Washer Method (6.1)

(Based on 6.1 #36) Use the washer method to find the volume of the solid obtained by rotating the region bounded by the curves $y = \tan(x)$, $x = 0$, and $y = 1$ around the x-axis. (Hint: Replace the tangent function in your integral using $\tan^2(x) = \sec^2(x) - 1$.)

Use the washer method to find the volume of the solid obtained by rotating the quadrilateral with vertices at (-2,0), (0,4), (2,4), and (2,0) around the line $x = -3$.

(See 6.1 #13-42 for more examples of solids of rotation.)

4. Volumes by Cylindrical Shells (6.2)

   (6.2 #2) Use the cylindrical shell method to find the volume of the solid obtained by rotating the region bounded by $y = 2 - \frac{x^2}{4}$, $x = 0$, $y = 0$, and $x = 2$ around the y-axis.

   Now check your answer by finding the volume of the same solid using the disk method.

(Based on 6.2 #16) Use the cylindrical shell method to find the volume of the solid obtained by rotating the region bounded by $y = -x$, $y = \sqrt{x}$, and $y = 2$ around the line $y = -1$.

Now check your answer by finding the volume of the same solid using the washer method.

(See 6.2 #1-12, 15-24 for more examples.)

5. Lengths of Parametric Curves in the Plane (6.3)

(6.3 #4) Find the length of the curve given by the parametric equations $x = \frac{t^2}{2}$ and $y = \frac{(2t+1)^{3/2}}{3}$ for $0 \leq t \leq 4$.

(6.3 #8) Find the length of the curve given by the parametric equations $x = e^t \cos t$ and $y = e^t \sin t$ for $0 \leq t \leq \pi$.

6. Lengths of Functions in the Plane (6.3)

(Based on 6.3 #10) Find the length of the curve $y = \frac{4}{3}x^{3/2}$ on the interval $0 \leq x \leq 4$.

(6.3 #18) Find the length of the curve $g(y) = \int_0^y \sqrt{\sec^4 t - 1}\, dt$ on the interval $-\pi/4 \leq y \leq \pi/4$.

(See 6.3 #1-18 for more examples.)

7. Work on a Spring (6.6)

   (Based on 6.6 #2) Recall Hooke's Law, which says that the force required to stretch/compress a spring x from its natural length is given by $F(x) = kx$. A particular spring's natural length is 10 inches. A force of 800 pounds stretches the spring to 14 inches. Find the force constant k of this spring.

   How much work is done in stretching the spring from 10 in. to 12 in.?

   How much work is done in compressing the spring from 10 in. to 8 in.?

   How much work is done in compressing the spring from 10 in. to 7 in.?

8. Work against Gravity (6.6)

(Based on 6.6 #8) A bag of sand originally weighing 144 lbs. is lifted at a constant rate. As it rises, sand leaks out at a rate of 4 lbs./ft. How much work is done in lifting this bag 10 feet?

(Based on Example 5 in 6.6) A conical tank of height 6m radius 2m stands on its point. A liquid weighing 10,000 N/m^3 is pumped into the tank from its point. How much work is done in completely filling the tank?

(See 6.6 1-10 for more Work examples. A calculator might be necessary.)