Calculus II Spring 2010 **Practice Test 4**

Your Name: **Instructor: Steven Clontz**

Draw a box around your final answer. You must show all work to receive credit.

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1. Find the interval and radius of convergence for $\sum_{n=1}^{\infty} \frac{(2x+3)^n}{\sqrt{n}}$.

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2. Find the Taylor Polynomial $P_2(x)$ of order 2 generated by $f(x) = \arctan(x)$ at $x = 1$.

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3. Find the Taylor Series generated by $f(x) = \ln(x)$ at $x = 1$.

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4. Recall that $\sin(x) = \lim_{n\to\infty} P_n(x) + \lim_{n\to\infty} R_n(x)$ where $\lim_{n\to\infty} P_n(x)$ is its Maclaurin Series and $R_n(x)$ is the error/remainder term given by Taylor's Formula.

   Show that the Maclaurin Series generated by $\sin(x)$

$$\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{(2k+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \ldots$$

   converges to $\sin(x)$ for any real number x by showing $\lim_{n\to\infty} R_n(x) = 0$.

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5. Express $\int \sin(e^x)dx$ as a power series.

6. Give a polynomial of degree 6 which approximates $f(x) = \dfrac{1}{\sqrt{1+x^2}}$.

7. For the polar point $(r, \theta) = (2, 5\pi/6)$,

- Give an equivalent polar point.

- Give the corresponding Cartesian (x, y) expression.

- Plot it on a graph.

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8. Sketch the graph of the polar equation $r^2 - r^2 \sin^2(\theta) = r \sin(\theta)$.

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9. Show that the polar graph of $r = 1 - \sin(2\theta)$ is rotationally symmetric about the origin.

10. Sketch the polar graph of $r = 1 - \sin(2\theta)$.

11. Show that the area of the region bounded by the polar equation $r = 1 - \sin(2\theta)$ is equal to the integral $\int_0^{2\pi} \left(\frac{1}{2} - \sin(2\theta) + \frac{1}{2}\sin^2(2\theta)\right) d\theta$.

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12. Show that the length of the perimeter of the region bounded by the polar equation $r = 1 - \sin(2\theta)$ is equal to the integral $\int_0^{2\pi} \sqrt{2 - 2\sin(2\theta) + 3\cos^2(2\theta)} d\theta$.
