

Your Name:

Answer Key

Instructor: Steven Clontz

Draw a box around your final answer. You must show all work to receive credit.

1. (15 pts) Find $\int (y+1) \sin(y) dy$.

$$\begin{aligned} \text{Let } u &= y+1 & v' &= \sin y \\ u' &= 1 & v &= -\cos y \end{aligned}$$

$$= -(y+1)\cos y + \int \cos y dy$$

$$= \boxed{-(y+1)\cos y + \sin y + C}$$

2. (10 pts) Find $\int e^w \sin(w) dw$.

$$\begin{aligned} \text{Let } u &= \sin w & v' &= e^w \\ u' &= \cos w & v &= e^w \end{aligned}$$

$$= e^w \sin w - \int e^w \cos w dw$$

$$\begin{aligned} \text{Let } u &= \cos w & v' &= e^w \\ u' &= -\sin w & v &= e^w \end{aligned}$$

$$= e^w \sin w - (e^w \cos w + \int e^w \sin w dw)$$

$$= e^w \sin w - e^w \cos w - \int e^w \sin w dw$$

$$2 \int = e^w \sin w - e^w \cos w + C$$

$$\int = \boxed{\frac{1}{2} e^w \sin w - \frac{1}{2} e^w \cos w + C}$$

3. (10 pts) Find $\int \sec^3(z) dz$.

$$= \int \sec z (\sec^2 z) dz$$

$$= \int \sec z (1 + \tan^2 z) dz$$

$$= \int \sec z dz + \int \sec z \tan^2 z dz$$

$$\text{Let } u = \tan z \quad v' = \sec z \tan z$$

$$u' = \sec^2 z \quad v = \sec z$$

$$= \ln|\sec z + \tan z| + \sec z \tan z - \int \sec^3 z dz$$

$$2 \int \sec^3 z dz = \ln|\sec z + \tan z| + \sec z \tan z + C$$

$$\int \sec^3 z dz = \boxed{\frac{1}{2} \ln|\sec z + \tan z| + \frac{1}{2} \sec z \tan z + C}$$

3. (10 pts)

4. (15 pts) Evaluate $\int_0^{\pi/4} \sqrt{1 + \cos(4\alpha)} d\alpha$

$$\cos^2(2\alpha) = \frac{1 + \cos(4\alpha)}{2}$$

$$2 \cos^2(2\alpha) = 1 + \cos(4\alpha)$$

$$= \int_0^{\pi/4} \sqrt{2 \cos^2(2\alpha)} d\alpha$$

$$= \int_0^{\pi/4} \sqrt{2} \cos(2\alpha) d\alpha$$

$$= \left[\sqrt{2} \frac{1}{2} \sin(2\alpha) \right]_0^{\pi/4}$$

$$= \frac{\sqrt{2}}{2} \sin\left(\frac{\pi}{2}\right) - \frac{\sqrt{2}}{2} \sin(0)$$

$$\boxed{\frac{\sqrt{2}}{2}}$$

5. (15 pts) Use the trigonometric formula $\sin(mx)\sin(nx) = \frac{\cos([m-n]x) - \cos([m+n]x)}{2}$

to find $\int 2\sin(4t)\sin(3t)dt$.

$$= \int \frac{\cos(t) - \cos(7t)}{2} dt$$

$$= \boxed{\sin t - \frac{1}{7}\sin 7t + C}$$

5. (15 pts)

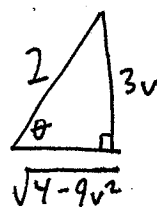
5. (15 pts)

6. (10 pts) Find $\int \frac{v}{\sqrt{4-9v^2}} dv$

Let $9v^2 = 4\sin^2\theta$
 $3v = 2\sin\theta$
 $v = \frac{2}{3}\sin\theta$
 $dv = \frac{2}{3}\cos\theta d\theta$

$$= \int \frac{2}{9}\sin\theta d\theta$$

$$= -\frac{2}{9}\cos\theta + C$$



$$\sin\theta = \frac{3v}{2} = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{4-9v^2}}{2}$$

$$= \int \frac{\frac{2}{3}\sin\theta}{\sqrt{4-4\sin^2\theta}} \cdot \frac{2}{3}\cos\theta d\theta$$

$$= \int \frac{4}{9} \frac{\sin\theta \cos\theta}{\sqrt{4\cos^2\theta}} d\theta$$

$$= \int \frac{2}{9} \frac{\sin\theta \cos\theta}{\cos\theta} d\theta$$

$$= \boxed{-\frac{1}{9}\sqrt{4-9v^2} + C}$$

(Letting $u = 4-9v^2$ would have been smarter, though...)

7. (15 pts) Expand $\frac{8s^4 + 8}{s^3 - 4s}$ using partial fractions.

$$8s + \frac{32s^2 + 8}{s^3 - 4s}$$

$$\begin{array}{r} s^3 - 4s \overline{) 8s^4 + 0s^3 + 0s^2 + 0s + 8} \\ \underline{-(8s^4 - 32s^2)} \\ 32s^2 + 8 \end{array}$$

$$8s + \frac{-2}{s} + \frac{17}{s-2} + \frac{17}{s+2}$$

$$\frac{32s^2 + 8}{s(s-2)(s+2)} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+2}$$

$$32s^2 + 8 = A(s-2)(s+2) + Bs(s+2) + Cs(s-2)$$

$$32s^2 + 8 = As^2 - 4A + Bs^2 + 2Bs + Cs^2 - 2Cs$$

$$32s^2 + 0s + 8 = (A+B+C)s^2 + (2B-2C)s + (-4A)$$

$$\begin{array}{l} s^2: 32 = A+B+C \\ s: 0 = 2B-2C \rightarrow 2B=2C \\ \text{Con: } 8 = -4A \end{array} \quad \begin{array}{l} A = -2 \\ 2B = 2C \\ 32 = -2 + B + B \\ 34 = 2B \\ 17 = B \\ 17 = C \end{array}$$

8. (10 pts) Evaluate $\int_e^\infty \frac{1}{\theta(\ln \theta)^2} d\theta$.

$$= \lim_{b \rightarrow \infty} \int_e^b \frac{1}{\theta(\ln \theta)^2} d\theta$$

$$= -0 + \frac{1}{1}$$

$$= \boxed{1}$$

$$\text{Let } u = \ln \theta$$

$$du = \frac{1}{\theta} d\theta$$

$$= \lim_{b \rightarrow \infty} \int_{\theta=e}^{\theta=b} \frac{1}{u^2} du$$

$$= \lim_{b \rightarrow \infty} \left[-\frac{1}{u} \right]_{\theta=e}^{\theta=b}$$

$$= \lim_{b \rightarrow \infty} \left[-\frac{1}{\ln \theta} \right]_e$$

$$= \lim_{b \rightarrow \infty} \left[-\frac{1}{\ln b} + \frac{1}{\ln e} \right]$$

9. (BONUS: 10 pts) Find the length of the curve given by the parametric equations $x = \cos(t^2 + \pi)$ and $y = \sin(t^2 + \pi)$ for $0 \leq t \leq \pi$.

$$\frac{dx}{dt} = -2t \sin(t^2 + \pi)$$

$$\frac{dy}{dt} = 2t \cos(t^2 + \pi)$$

$$s = \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$s = \int_0^{\pi} \sqrt{4t^2 \sin^2(t^2 + \pi) + 4t^2 \cos^2(t^2 + \pi)} dt$$

$$s = \int_0^{\pi} \sqrt{4t^2} \sqrt{\sin^2(t^2 + \pi) + \cos^2(t^2 + \pi)} dt$$

$$s = \int_0^{\pi} 2t dt$$

$$s = [t^2]_0^{\pi}$$

$$s = \boxed{\pi^2}$$

10. (BONUS: 5 pts) Find $\int \sin(\ln x) dx$. (Hint: Multiply by $\frac{e^{\ln x}}{x}$, use substitution on $\ln x$, then use the result from question 2.)

$$= \int e^{\ln x} \sin(\ln x) \frac{1}{x} dx$$

$$\text{Let } w = \ln x$$

$$dw = \frac{1}{x} dx$$

$$= \int e^w \sin w dw$$

$$= \frac{1}{2} e^w \sin w - \frac{1}{2} e^w \cos w + C$$

$$= \frac{1}{2} e^{\ln x} \sin(\ln x) - \frac{1}{2} e^{\ln x} \cos(\ln x) + C$$

$$= \frac{1}{2} x \sin(\ln x) - \frac{1}{2} x \cos(\ln x) + C$$

10/10/10

Dear Sir,
I am writing to you regarding the matter of the
contract for the supply of goods to the
Government of India. I am pleased to
hear that you are interested in
supplying goods to the Government of
India. I am sure that your goods will
be of high quality and will meet the
requirements of the Government of India.
I am sure that you will be able to
supply the goods in a timely manner.
I am sure that you will be able to
supply the goods in a timely manner.
I am sure that you will be able to
supply the goods in a timely manner.

Yours faithfully,
[Signature]

I am sure that you will be able to
supply the goods in a timely manner.
I am sure that you will be able to
supply the goods in a timely manner.
I am sure that you will be able to
supply the goods in a timely manner.