Calculus II Spring 2010 Test 2

Your Name: Answer Key Instructor: Steven Clontz

Draw a box around your final answer. You must show all work to receive credit.

---

1. (15 pts) Find $\int (y+1)\sin(y)dy$.

Let $u = y+1 \quad v' = \sin y$

$u' = 1 \quad v = -\cos y$

$= -(y+1)\cos y + \int \cos y\, dy$

$= \boxed{-(y+1)\cos y + \sin y + C}$

---

2. (10 pts) Find $\int e^w \sin(w)dw$.

Let $u = \sin w \quad v' = e^w$

$u' = \cos w \quad v = e^w$

$= e^w \sin w - \int e^w \cos w\, dw$

Let $u = \cos w \quad v' = e^w$

$u' = -\sin w \quad v = e^w$

$= e^w \sin w - \left(e^w \cos w + \int e^w \sin w\, dw\right)$

$= e^w \sin w - e^w \cos w - \int e^w \sin w\, dw$

$2\int \sim = e^w \sin w - e^w \cos w + C$

$\int \sim = \boxed{\tfrac{1}{2} e^w \sin w - \tfrac{1}{2} e^w \cos w + C}$

3. (10 pts) Find $\int \sec^3(z)dz$.

$$= \int \sec z \,(\sec^2 z)\,dz$$

$$= \int \sec z \,(1 + \tan^2 z)\,dz$$

$$= \int \sec z\,dz + \int \sec z \tan^2 z\,dz$$

Let $u = \tan z \quad v' = \sec z \tan z$

$u' = \sec^2 z \quad v = \sec z$

$$= \ln|\sec z + \tan z| + \sec z \tan z - \int \sec^3 z\,dz$$

$$2\int — = \ln|\sec z + \tan z| + \sec z \tan z + C$$

$$\int — = \boxed{\tfrac{1}{2}\ln|\sec z + \tan z| + \tfrac{1}{2}\sec z \tan z + C}$$

---

4. (15 pts) Evaluate $\int_0^{\pi/4} \sqrt{1+\cos(4\alpha)}\,d\alpha$

$$\cos^2(2\alpha) = \frac{1+\cos(4\alpha)}{2}$$

$$2\cos^2(2\alpha) = 1 + \cos(4\alpha)$$

$$= \int_0^{\pi/4} \sqrt{2\cos^2(2\alpha)}\,d\alpha$$

$$= \int_0^{\pi/4} \sqrt{2}\cos(2\alpha)\,d\alpha$$

$$= \left[\sqrt{2}\,\tfrac{1}{2}\sin(2\alpha)\right]_0^{\pi/4}$$

$$= \tfrac{\sqrt{2}}{2}\sin(\tfrac{\pi}{2}) - \cancel{\tfrac{\sqrt{2}}{2}\sin(0)}$$

$$\boxed{\tfrac{\sqrt{2}}{2}}$$

---

5. (15 pts) Use the trigonometric formula $\sin(mx)\sin(nx) = \dfrac{\cos([m-n]x) - \cos([m+n]x)}{2}$ to find $\int 2\sin(4t)\sin(3t)dt$.

$$= \int \cancel{2}\,\frac{\cos(t) - \cos(7t)}{\cancel{2}}\,dt$$

$$= \boxed{\sin t - \tfrac{1}{7}\sin 7t + C}$$

---

6. (10 pts) Find $\int \dfrac{v}{\sqrt{4-9v^2}}dv$

Let $9v^2 = 4\sin^2\theta$

$3v = 2\sin\theta$

$v = \tfrac{2}{3}\sin\theta$

$dv = \tfrac{2}{3}\cos\theta\, d\theta$

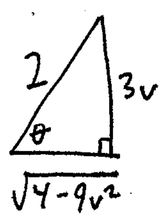
$$= \int \frac{\tfrac{2}{3}\sin\theta}{\sqrt{4-4\sin^2\theta}}\,\tfrac{2}{3}\cos\theta\, d\theta$$

$$= \int \tfrac{4}{9}\,\frac{\sin\theta\cos\theta}{\sqrt{4\cos^2\theta}}\,d\theta$$

$$= \int \tfrac{\cancel{4}^{\,2}}{9}\,\frac{\sin\theta\,\cancel{\cos\theta}}{\cancel{2}\,\cancel{\cos\theta}}\,d\theta$$

$$= \int \tfrac{2}{9}\sin\theta\, d\theta$$

$$= -\tfrac{2}{9}\cos\theta + C$$



$\sin\theta = \dfrac{3v}{2} = \dfrac{opp}{hyp}$

$\cos\theta = \dfrac{adj}{hyp} = \dfrac{\sqrt{4-9v^2}}{2}$

$$= \boxed{-\tfrac{1}{9}\sqrt{4-9v^2} + C}$$

(Letting $u = 4-9v^2$ would have been smarter, though...)

---

7. (15 pts) Expand $\dfrac{8s^4+8}{s^3-4s}$ using partial fractions.

$$\begin{array}{r} 8s + \frac{32s^2+8}{s^3-4s} \\ s^3-4s \overline{)\,8s^4+0s^3+0s^2+0s+8} \\ -(8s^4 \quad -32s^2) \\ \hline 32s^2 \quad +8 \end{array}$$

$$\boxed{8s + \frac{-2}{s} + \frac{17}{s-2} + \frac{17}{s+2}}$$

$$\frac{32s^2+8}{s^3-4s} = \frac{32s^2+8}{s(s-2)(s+2)} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+2}$$

$$32s^2+8 = A(s-2)(s+2) + Bs(s+2) + Cs(s-2)$$

$$32s^2+8 = As^2 - 4A + Bs^2 + 2Bs + Cs^2 - 2Cs$$

$$32s^2+0s+8 = (A+B+C)s^2 + (2B-2C)s + (-4A)$$

s^2: $32 = A+B+C$

s: $0 = 2B-2C \rightarrow 2B = 2C$

Con: $8 = -4A$

$A = -2$

$32 = -2+B+B$

$34 = 2B$

$17 = B$

$17 = C$

8. (10 pts) Evaluate $\displaystyle\int_e^\infty \frac{1}{\theta(\ln\theta)^2}\,d\theta$.

$$= \lim_{b\to\infty} \int_e^b \frac{1}{\theta(\ln\theta)^2}\,d\theta$$

Let $u = \ln\theta$

$du = \frac{1}{\theta}\,d\theta$

$$= \lim_{b\to\infty} \int_{\theta=e}^{\theta=b} \frac{1}{u^2}\,du$$

$$= \lim_{b\to\infty} \left[-\frac{1}{u}\right]_{\theta=e}^{\theta=b}$$

$$= \lim_{b\to\infty} \left[-\frac{1}{\ln\theta}\right]_e^b$$

$$= \lim_{b\to\infty} \left[-\frac{1}{\ln b} + \frac{1}{\ln e}\right]$$

$$= -0 + \frac{1}{1}$$

$$= \boxed{1}$$

9. (BONUS: 10 pts) Find the length of the curve given by the parametric equations $x = \cos(t^2+\pi)$ and $y = \sin(t^2+\pi)$ for $0 \le t \le \pi$.

$$\frac{dx}{dt} = -2t\sin(t^2+\pi)$$

$$\frac{dy}{dt} = 2t\cos(t^2+\pi)$$

$$s = \int_0^\pi \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$s = \int_0^\pi \sqrt{4t^2\sin^2(t^2+\pi) + 4t^2\cos^2(t^2+\pi)}\, dt$$

$$s = \int_0^\pi \sqrt{4t^2}\sqrt{\sin^2(t^2+\pi) + \cos^2(t^2+\pi)}\, dt$$

$$s = \int_0^\pi 2t\cancel{\sqrt{1}}\, dt$$

$$s = \left[t^2\right]_0^\pi$$

$$s = \boxed{\pi^2}$$

---

10. (BONUS: 5 pts) Find $\int \sin(\ln x)dx$. (Hint: Multiply by $\frac{e^{\ln x}}{x}$, use substitution on $\ln x$, then use the result from question 2.)

$$= \int e^{\ln x} \sin(\ln x) \frac{1}{x} dx$$

Let $w = \ln x$

$dw = \frac{1}{x} dx$

$$= \int e^w \sin w \, dw$$

$$= \tfrac{1}{2} e^w \sin w - \tfrac{1}{2} e^w \cos w + C$$

$$\cancel{\tfrac{1}{2} e^{\ln x} \sin(\ln x) - \tfrac{1}{2} e^{\ln x} \cos(\ln x) + C}$$

$$= \boxed{\tfrac{1}{2} x \sin(\ln x) - \tfrac{1}{2} x \cos(\ln x) + C}$$
