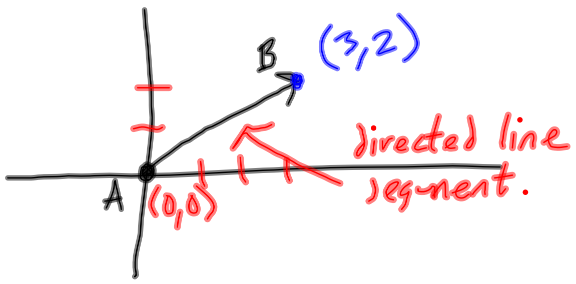
10.2 Vectors

Scalar: any real number / magnitude

(Idea: a scale gives a value, but not direction.)

Vector: stores information about magnitude AND direction.



Book Defn)

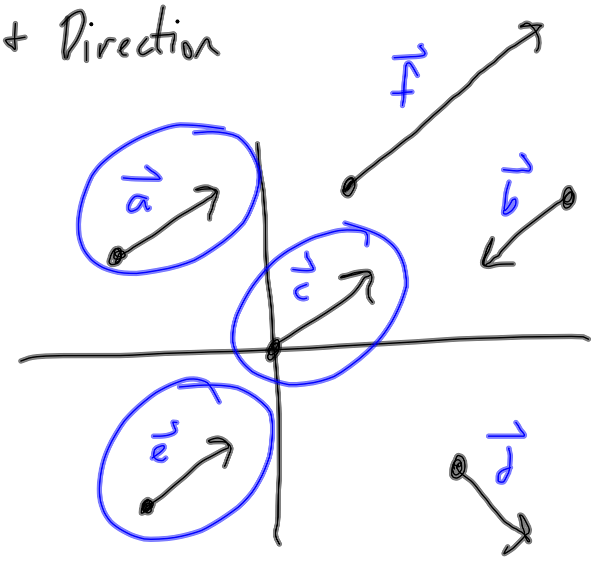
A vector is a directed line segment $\overrightarrow{AB}$.

initial point

terminal point

Its length is given by $|\overrightarrow{AB}|$.

Two vectors are equal if the have the same length + Direction

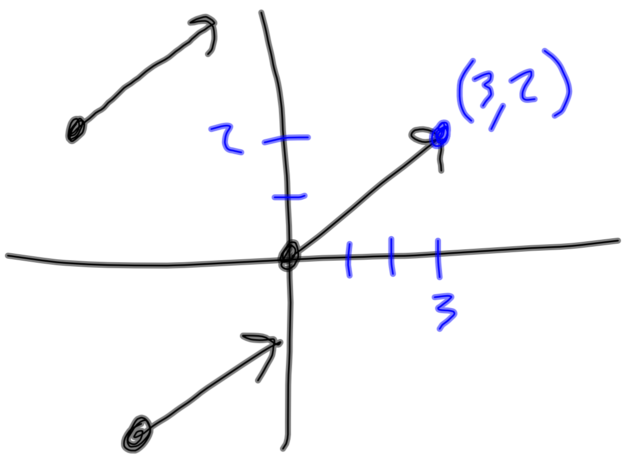


$\vec{a}, \vec{c}, \vec{e}$ are equal.

Typically, we often think of vectors as having their initial point on the origin.

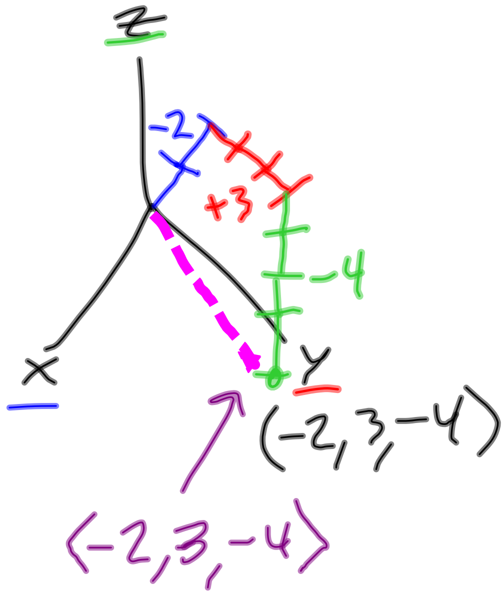
How vectors are usually defined:

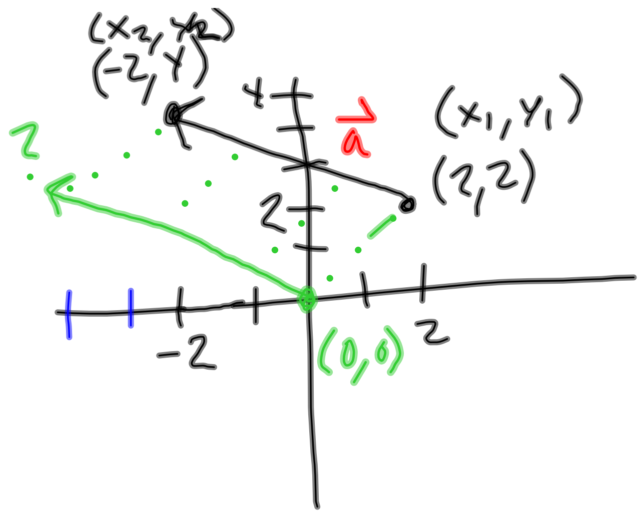
2-Dimensions



Each of these vectors is equal to the vector $\langle 3,2 \rangle$.

3-Dimensions





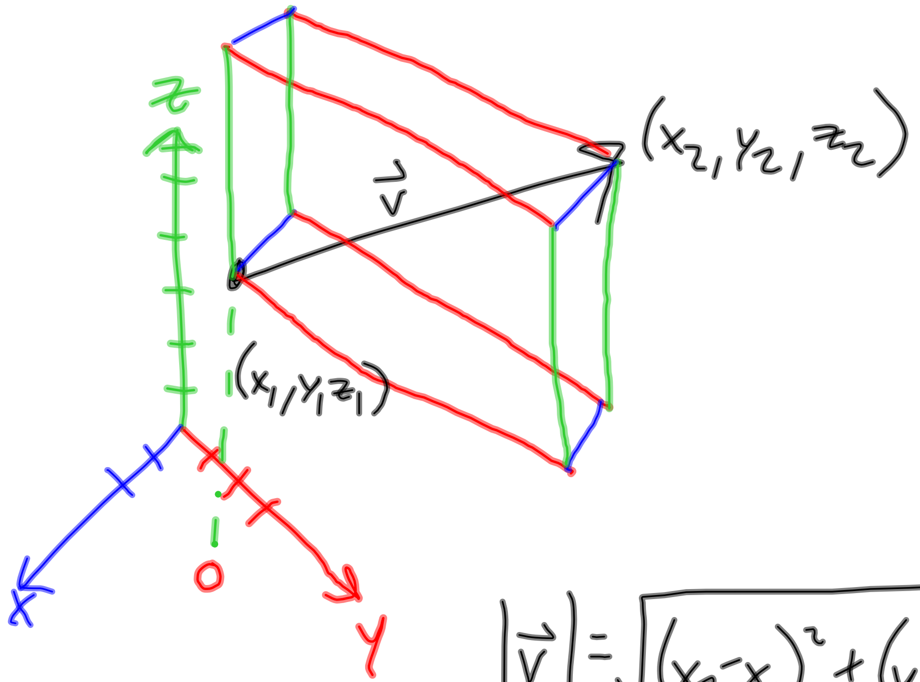
What is a name (component form) for the vector $\vec{a}$?

$$\langle -2-(2),\ 4-2\rangle = \langle -4, 2\rangle$$

Formula to find the component form for a vector with initial point (x_1, y_1, z_1) + terminal point (x_2, y_2, z_2)

$$\langle x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1\rangle$$

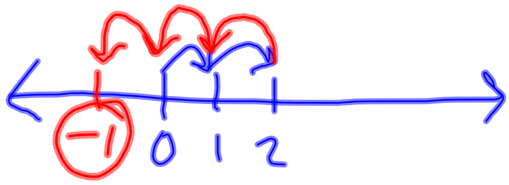
How to find the length of a vector.



$$|\vec{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Operations of Scalars

$2+(-3)=-1$

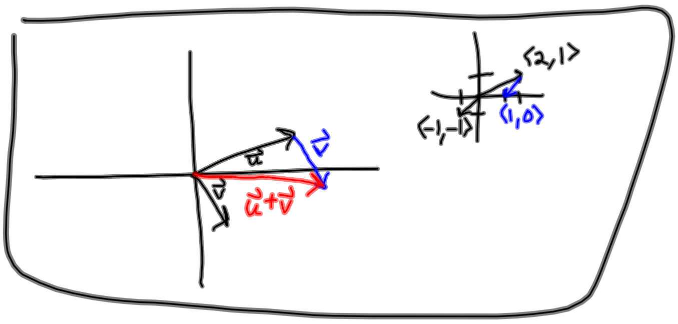


$7(-2)=-14$

Operations of Vectors

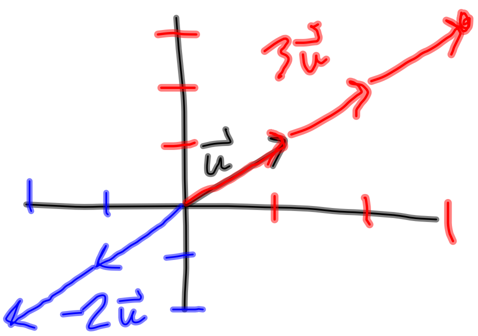
$\langle u_1, u_2\rangle + \langle v_1, v_2\rangle = \langle u_1+v_1 \, , \, u_2+v_2\rangle$

OR $\vec{u} + \vec{v}$



Scalar

$k\langle u_1, u_2\rangle = \langle ku_1, ku_2\rangle$



Some properties for scalar addition + multiplication

$a+b=b+a \qquad a(1)=a$

$a+0=a \qquad a(b+c)=ab+ac$

Vector Operation Properties

1. $\vec{u}+\vec{v}=\vec{v}+\vec{u}$
2. $(\vec{u}+\vec{v})+\vec{w}=\vec{u}+(\vec{v}+\vec{w})$
3. $\vec{u}+\vec{0}=\vec{u}$
4. $\vec{u}+(-\vec{u})=\vec{0}$
5. $0\vec{u}=\vec{0}$
6. $1\vec{u}=\vec{u}$
7. $a(b\vec{u})=(ab)\vec{u}$
8. $a(\vec{u}+\vec{v})=a\vec{u}+a\vec{v}$
9. $(a+b)\vec{u}=a\vec{u}+b\vec{u}$

$\vec{0}=\langle 0,0\rangle$ or $\langle 0,0,0\rangle$ etc.

$-\vec{u}=(-1)\vec{u}$

10. $(a+b)(\vec{u}+\vec{v})$
$=a\vec{u}+b\vec{u}+a\vec{v}+b\vec{v}$

Prove 8

$$\begin{aligned}
a(\vec{u}+\vec{v}) &= a(\langle u_1,u_2\rangle+\langle v_1,v_2\rangle)\\
&= a(\langle u_1+v_1,\, u_2+v_2\rangle)\\
&= \langle a(u_1+v_1),\, a(u_2+v_2)\rangle\\
&= \langle au_1+av_1,\, au_2+av_2\rangle\\
&= \langle au_1,\, au_2\rangle+\langle av_1,\, av_2\rangle\\
&= a\underbrace{\langle u_1,u_2\rangle}+a\underbrace{\langle v_1,v_2\rangle}\\
&= a\vec{u} \quad + \quad a\vec{v}
\end{aligned}$$

Unit Vectors:

		Two Dimensions	Three Dim.
x	$\vec{i}$	$\langle 1,0\rangle$	$\langle 1,0,0\rangle$
y	$\vec{j}$	$\langle 0,1\rangle$	$\langle 0,1,0\rangle$
z	$\vec{k}$		$\langle 0,0,1\rangle$

Example:

$$\langle -3,0,7\rangle = \langle -3,0,0\rangle + \langle 0,0,7\rangle$$
$$= -3\langle 1,0,0\rangle + 7\langle 0,0,1\rangle$$
$$= -3\vec{i} + 7\vec{k}$$

$$\langle a,b,c\rangle = a\vec{i} + b\vec{j} + c\vec{k}$$