Series 8.2 Examples

Do the following infinite sums converge/diverge?
If they converge, what is the value of the sum?

$$1-2+4-8+16-32+\dots$$

Diverges b/c

$\langle 1, -2, 4, -8, 16, -32, \dots \rangle$ diverges

$$\frac{5}{1\cdot 2}+\frac{5}{2\cdot 3}+\frac{5}{3\cdot 4}+\dots$$

b/c $\frac{1}{i(i+1)} = \frac{1}{i} - \frac{1}{i+1}$

$$= \sum_{i=1}^{\infty} \frac{5}{i(i+1)} = \sum_{i=1}^{\infty} \frac{5}{i} - \frac{5}{i+1}$$

$$= \left(\frac{5}{1} - \cancel{\frac{5}{2}}\right) + \left(\cancel{\frac{5}{2}} - \cancel{\frac{5}{3}}\right) + \left(\cancel{\frac{5}{3}} - \cancel{\frac{5}{4}}\right) + \left(\cancel{\frac{5}{4}} \dots\right.$$

$$= \boxed{5}$$ so converges

$$\sum_{k=0}^{\infty}\left(\frac{2^{k+1}}{5^{k}}\right)$$

$$2^n = \underbrace{\overbrace{2}^{}}_{1}\cdot\underbrace{2 \cdot 2 \cdots 2}_{n-1}$$

$$\overbrace{2 \cdot 2 \cdot 2 \cdots 2}^{n}$$

-1 $\quad$ $+1$ $\quad$ -1

$$= \sum_{n=1}^{\infty} \frac{2^n}{5^{n-1}} = \sum_{n=1}^{\infty} \frac{2(2^{n-1})}{5^{n-1}}$$

$$= \sum_{n=1}^{\infty} 2\left(\frac{2}{5}\right)^{n-1}$$

$$= \frac{2}{1-2/5} = \frac{2}{3/5} = \boxed{10/3}$$

$$\sum_{n=2}^{\infty} \left(\frac{1}{2^{1/n}} - \frac{1}{2^{\left(\frac{1}{n+1}\right)}} \right)$$

$$= \left(\frac{1}{2^{1/2}} - \cancel{\frac{1}{2^{1/3}}} \right) + \left(\cancel{\frac{1}{2^{1/3}}} - \cancel{\frac{1}{2^{1/4}}} \right) + \left(\cancel{\frac{1}{2^{1/4}}} - \cancel{\frac{1}{2^{1/5}}} \right) + \cdots$$

$$= \boxed{\frac{1}{\sqrt{2}}} \quad \text{converges}$$

$$\sum_{i=1}^{\infty} \frac{2+\cos(\pi i)}{5^i} = \sum_{i=1}^{\infty} \frac{2}{5^i} + \frac{\cos(\pi i)}{5^i}$$

$$= \sum_{i=1}^{\infty} \frac{2}{5^i} + \sum_{i=1}^{\infty} \frac{\cos(\pi i)}{5^i}$$

$$= \sum_{i=1}^{\infty} 2\frac{1}{5(5^{i-1})} + \sum_{i=1}^{\infty} \frac{(-1)^i}{5^i}$$

$$= \sum_{i=1}^{\infty} \frac{2}{5}\left(\frac{1}{5}\right)^{i-1} + \sum_{i=1}^{\infty} \left(-\frac{1}{5}\right)^i$$

$$= \frac{2/5}{1-1/5} + \sum_{i=1}^{\infty} \left(-\frac{1}{5}\right)\left(-\frac{1}{5}\right)^{i-1}$$

$$= \frac{2/5}{4/5} + \frac{-1/5}{1+1/5}$$

$$= 1/2 - \frac{1/5}{6/5}$$

$$= 1/2 - 1/6$$

$$= \boxed{1/3}$$

$$\sum_{i=3}^{\infty} \frac{i^i}{i!} = \frac{3^3}{3!} + \frac{4^4}{4!} + \frac{5^5}{5!} + \cdots$$

$$= \frac{27}{6} + \frac{64}{24} + \frac{5^5}{120} + \cdots$$

$$\geq 1 + 1 + 1 + \cdots = \infty$$

diverge

OR $\left\langle \frac{27}{6}, \frac{64}{24}, \frac{5^5}{120}, \ldots \right\rangle \to \infty$

so $\sum \frac{i^i}{i!}$ diverges