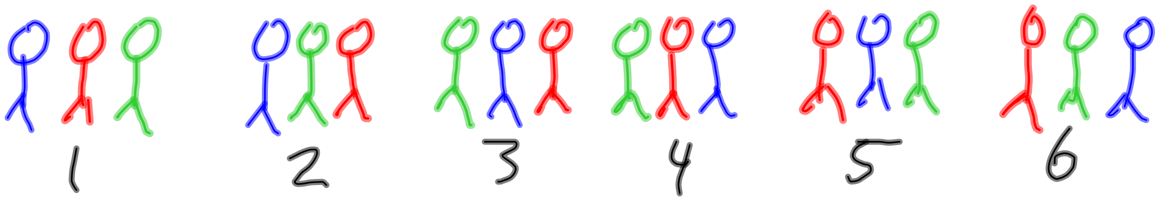
8.1 Sequences

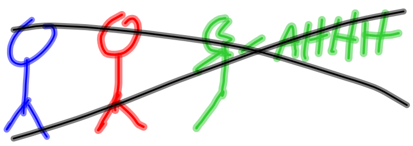
Sometimes we just want to investigate functions of whole numbers.

Example: Let s(n) represent the number of ways to order n people in a row.

s(1) = 1
s(2) = 2
s(3) = 6



Of course, s(2.5) doesn't really make sense.



Definition: a **sequence** is a function whose domain is the positive integers.

Ways to represent a sequence:

* Like a function $a(n) = a_n$

Examples

$a_n = \sqrt{n}$

$b_n = (-1)^{n+1} \frac{1}{n}$

$c_n = \frac{n-1}{n}$

$d_n = (-1)^{n+1}$

* By listing terms

$a_n = \langle \sqrt{1}, \sqrt{2}, \sqrt{3}, \ldots, \sqrt{n}, \ldots \rangle$

$b_n = \langle \frac{1}{1}, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \ldots, (-1)^{n+1}\frac{1}{n}, \ldots \rangle$

$c_n = \langle 0, \frac{1}{2}, \frac{2}{3}, \ldots, \frac{n-1}{n}, \ldots \rangle$

$d_n = \langle 1, -1, 1, -1, \ldots, (-1)^{n+1}, \ldots \rangle$

If the pattern of the sequence is obvious, we sometimes just write the first few terms.

$b_n = \langle \frac{1}{1}, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \ldots \rangle$

We also sometimes represent them like this:

$a_n = \langle \sqrt{n} \rangle_{n=1}^{\infty}$

Ways to define a sequence

Same as a function: explicitly

$$a_n = \langle \sqrt{n} \rangle_{n=1}^{\infty} = \sqrt{n}$$

etc.

Recursively

First, you define some initial terms in the sequence

$$f_1 = 1 \qquad f_2 = 1$$

Then, you define all future terms based on previous terms

$$f_{n+2} = f_n + f_{n+1}$$

In this example, we get:

(BTW, this is the Fibonacci sequence)

$$f_n = \langle 1, 1, 2, 3, 5, 8, \ldots \rangle$$

$$f_{\underset{(3)}{1+2}} = f_1 + f_{1+1} = 1 + 1 = 2 \qquad f_4 = f_2 + f_3 = 1 + 2 = 3$$

Another example:

$$!_1 = 1, \quad !_{n+1} = (!_n)(n+1)$$

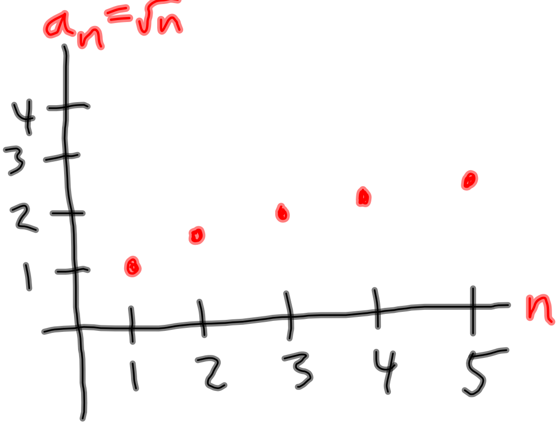
$$!_n = \langle 1, 2, 6, 24, 120, 720, \ldots \rangle$$

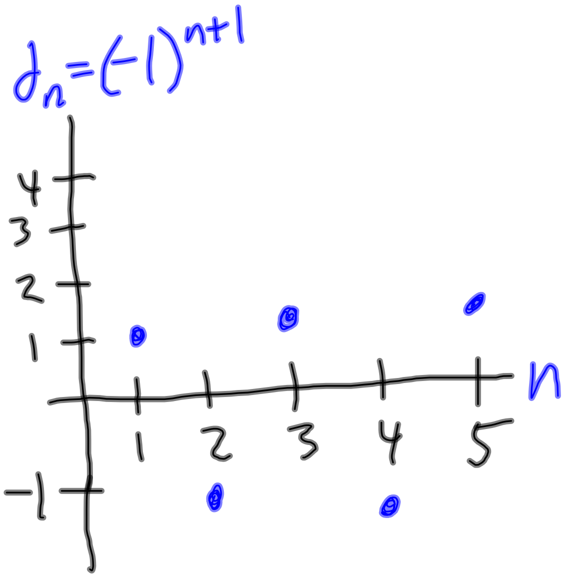
(This is a factorial sequence, which we usually write as n!)

$$0! = 1$$

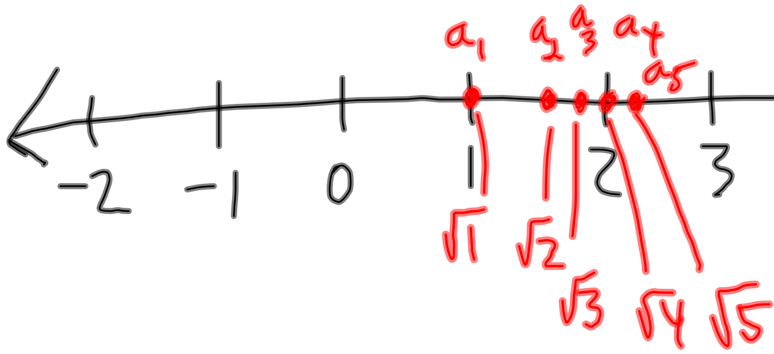
Ways to graph a sequence:

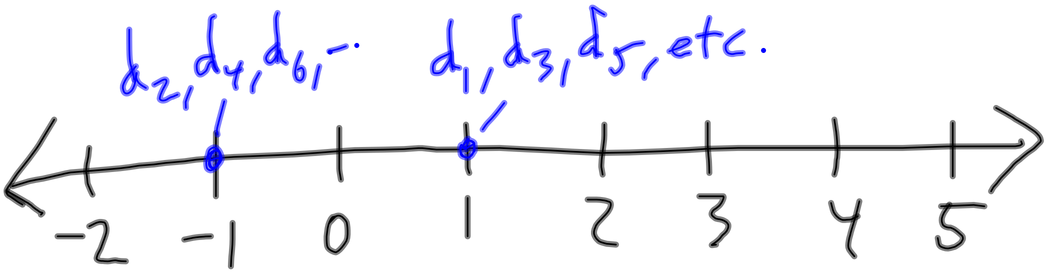
Like a function:





I can also graph on a single number line.





Limits of sequences:

We say a sequence a_n **converges** if it gets arbitrarily close to a single number L for successive terms in the sequence.

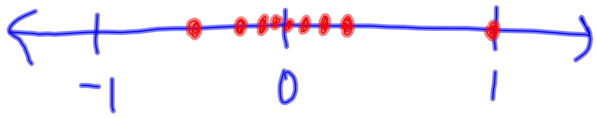
We say: $a_n \to L$ and $\lim_{n\to\infty} a_n = L$

If the sequence doesn't converge to a single number, we say it **diverges.**

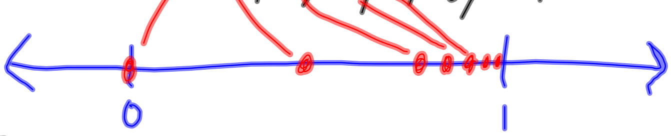
If the sequence diverges and is getting arbitrarily large for successive terms, then we say $a_n \to \infty$ and $\lim_{n\to\infty} a_n = \infty$

Ditto for $-\infty$

- $a_n = \sqrt{n}$
  - ☐ diverges
  - ☐ $a_n \to \infty$ or $\lim_{n\to\infty} a_n = \infty$
- $b_n = (-1)^{n+1}\frac{1}{n} = \langle 1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \ldots \rangle$



  - ☐ converges
  - ☐ $b_n \to 0$ or $\lim_{n\to\infty} b_n = 0$
- $c_n = \frac{n-1}{n} = \langle 0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots \rangle$



  - ☐ Converges
  - ☐ $c_n \to 1$ or $\lim_{n\to\infty} c_n = 1$
- $d_n = (-1)^{n+1} = \langle 1, -1, 1, -1, \ldots \rangle$
  - ☐ diverges (doesn't get close to SINGLE number)
  - ☐ $\cancel{d_n \to \infty}$ $\lim_{n\to\infty} d_n$ DNE