Binomial Series

$$(1+x)^m = \sum_{k=0}^{\infty} \frac{(m)(m-1)\cdots(m-k+1)}{k!} x^k$$

$$= \sum_{k=0}^{\infty} \frac{\prod_{i=0}^{k-1}(m-i)}{k!} x^k = \sum_{k=0}^{\infty} \binom{m}{k} x^k$$

$$= \frac{\prod_{i=0}^{-1}(m-i)}{0!} x^0 + \frac{\prod_{i=0}^{0}(m-i)}{1!} x^1 + \frac{\prod_{i=0}^{1}(m-i)}{2!} x^2 + \cdots$$

$$= \frac{1}{1}(1) + \frac{m}{1} x + \frac{m(m-1)}{2} x^2 + \cdots$$

exponent

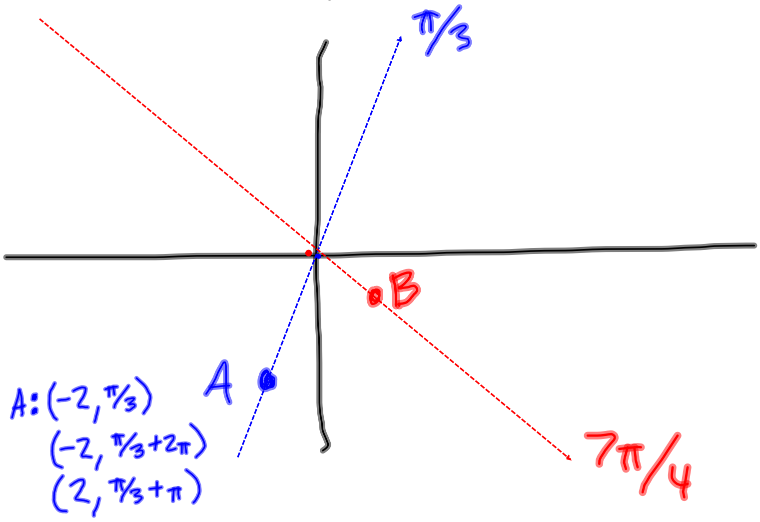
$$= 1 + mx + \frac{m(m-1)}{2!} x^2 + \frac{m(m-1)(m-2)}{3!} x^3$$

$$+ \frac{m(m-1)(m-2)(m-3)}{4!} x^4 + \cdots$$

Polar Coordinates

$B: (1, 7\pi/4)$
$(1, -\pi/4)$
$(-1, 3\pi/4)$

$P(r, \theta)$ represents the point on a coordinate plane which is the directed distance r in the direction of the angle θ from the origin. (Where $\theta = 0$ is the direction of the positive x-axis.)

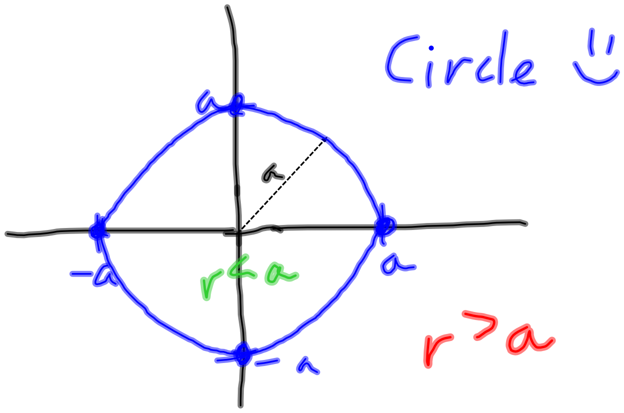


$A: (-2, \pi/3)$
$(-2, \pi/3 + 2\pi)$
$(2, \pi/3 + \pi)$

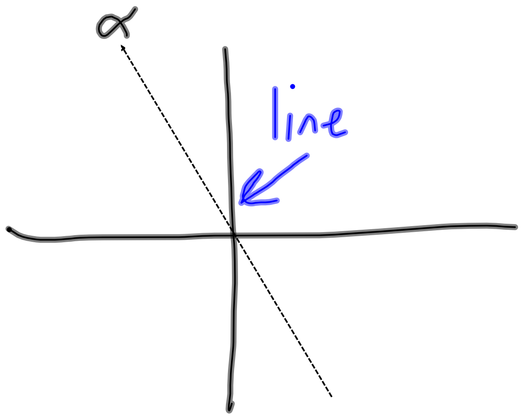
A point (r, θ) is at the same position as $(r, \theta + 2\pi k)$ for any integer k and $(-r, \theta + \pi + 2\pi k)$ for any integer k.

Two basic polar graphs

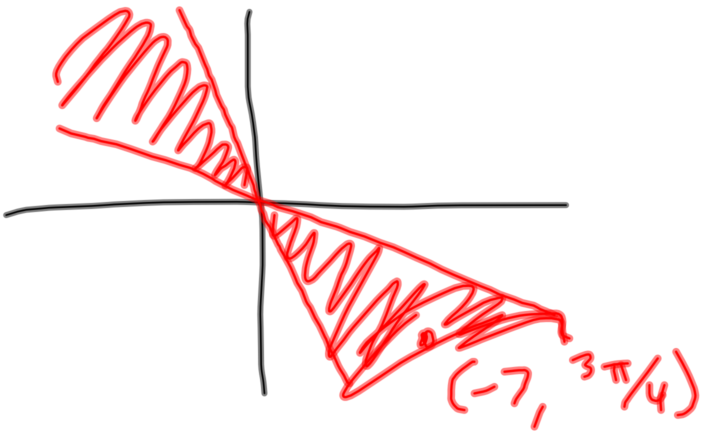
$r = a \leftarrow$ some constant



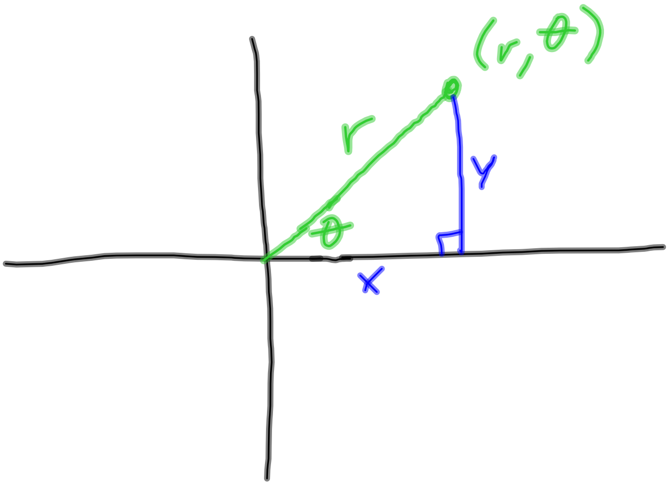
$\theta = \alpha \leftarrow$ some constant



Ex: Graph $2\pi/3 \le \theta \le 5\pi/6$



Relating Polar and Cartesian Coordinates



$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$

$\boxed{x = r\cos\theta}$

$\sin\theta = \frac{y}{r}$

$\boxed{y = r\sin\theta}$

$\boxed{x^2 + y^2 = r^2}$

What does the polar graph $r = -4\sec\theta$ look like?

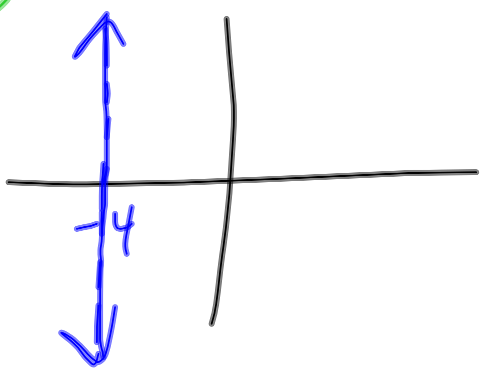
$r = -4\sec\theta$

$r = -\frac{4}{\cos\theta}$

$r\cos\theta = -4$

$x = -4$

Vertical line at $x = -4$



$$r^2 = 4r\cos\theta$$

$$x^2 + y^2 = 4x$$

$\div 2$, then square

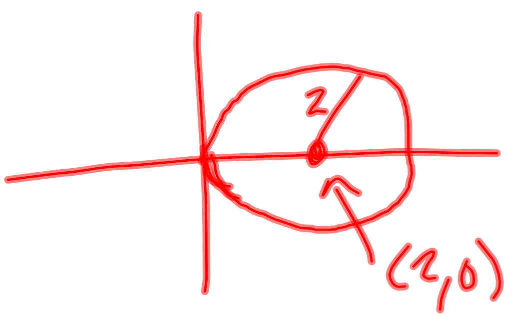
$$x^2 - 4x + 4 \quad + y^2 = 0 + 4$$

$$(x-2)^2 + y^2 = 4$$

x of cent. y of cent.

Circle with radius $\sqrt{4} = 2$.

and center @ $(2,0)$



$$r = \frac{4}{2\cos\theta - \sin\theta}$$

$$r(2\cos\theta - \sin\theta) = 4$$

$$2\underline{r\cos\theta} - \underline{r\sin\theta} = 4$$

$$2x - y = 4$$

$$y = 2x - 4$$