Taylor ~~Series~~ Polynomial of Order 3 @ $x=a$
generated by $f(x)$.

$$\sum_{k=0}^{\cancel{\infty}\,3} \frac{f^{(k)}(a)}{k!}(x-a)^{k}$$

$$= \frac{f^{(0)}(a)}{\cancel{0!}}\cancel{(x-a)^0} + \frac{f^{(1)}(a)}{1!}(x-a)^1 + \frac{f^{(2)}(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3$$

Area bounded by the graph of a polar equation and $\alpha \leq \theta \leq \beta$:

$$\int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta$$

How can I get a formula for lengths of polar curves?

Observation: Polar equations are basically parametric equations with variable θ for x & y.

$$x = r\cos\theta \qquad\qquad x(\theta) = f(\theta)\cos\theta$$
$$\Longrightarrow$$
$$y = r\sin\theta \qquad\qquad y(\theta) = f(\theta)\sin\theta$$

Where $f(\theta) = r$

Recall the formula for the length of a curve defined by parametric equations for ~~a <= t <= b~~ $\alpha < \theta \le \beta$

$$L = \int_{\cancel{a}\ \alpha}^{\cancel{b}\ \beta} \sqrt{\left(\frac{dx}{d\cancel{t}\,\theta}\right)^2 + \left(\frac{dy}{d\cancel{t}\,\theta}\right)^2}\ \cancel{dt}\ d\theta$$

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\ d\theta$$

$$= \int_{\alpha}^{\beta} \sqrt{\left(\frac{d}{d\theta}[x]\right)^2 + \left(\frac{d}{d\theta}[y]\right)^2}\ d\theta$$

$$= \int_{\alpha}^{\beta} \sqrt{\left(\frac{d}{d\theta}[r\cos\theta]\right)^2 + \left(\frac{d}{d\theta}[r\sin\theta]\right)^2}\ d\theta$$

$$= \int_{\alpha}^{\beta} \sqrt{\left(\frac{dr}{d\theta}\cos\theta - r\sin\theta\right)^2 + \left(\frac{dr}{d\theta}\sin\theta + r\cos\theta\right)^2}\ d\theta$$

$$= \int_{\alpha}^{\beta} \sqrt{\left(\frac{dr}{d\theta}^2\cos^2\theta - \cancel{2r\frac{dr}{d\theta}\cos\theta\sin\theta} + r^2\sin^2\theta\right) + \left(\frac{dr}{d\theta}^2\sin^2\theta + \cancel{2\frac{dr}{d\theta}r\sin\theta\cos\theta} + r^2\cos^2\theta\right)}\ d\theta$$

$$= \int_{\alpha}^{\beta} \sqrt{\frac{dr}{d\theta}^2\cancel{(\cos^2\theta + \sin^2\theta)} + r^2\cancel{(\sin^2\theta + \cos^2\theta)}}\ d\theta$$

$$L = \boxed{\int_{\alpha}^{\beta} \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\ d\theta}$$

Find the length of the cardiod $r = 1 - \cos\theta$.
$(0 \le \theta \le 2\pi)$

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\, d\theta$$

$\frac{dr}{d\theta} = \sin\theta$

$$= \int_0^{2\pi} \sqrt{(\sin\theta)^2 + (1-\cos\theta)^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{\sin^2\theta + 1 - 2\cos\theta + \cos^2\theta}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{\underline{\sin^2\theta + \cos^2\theta} + 1 - 2\cos\theta}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{1 + 1 - 2\cos\theta}\, d\theta$$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$

$4\sin^2\frac{\theta}{2} = 2 - 2\cos\theta$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos\theta}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{4\sin^2\frac{\theta}{2}}\, d\theta$$

$$= \int_0^{2\pi} 2\left|\sin\frac{\theta}{2}\right| d\theta$$

$\sin\frac{\theta}{2}$ is positive for $0 \le \theta \le 2\pi$

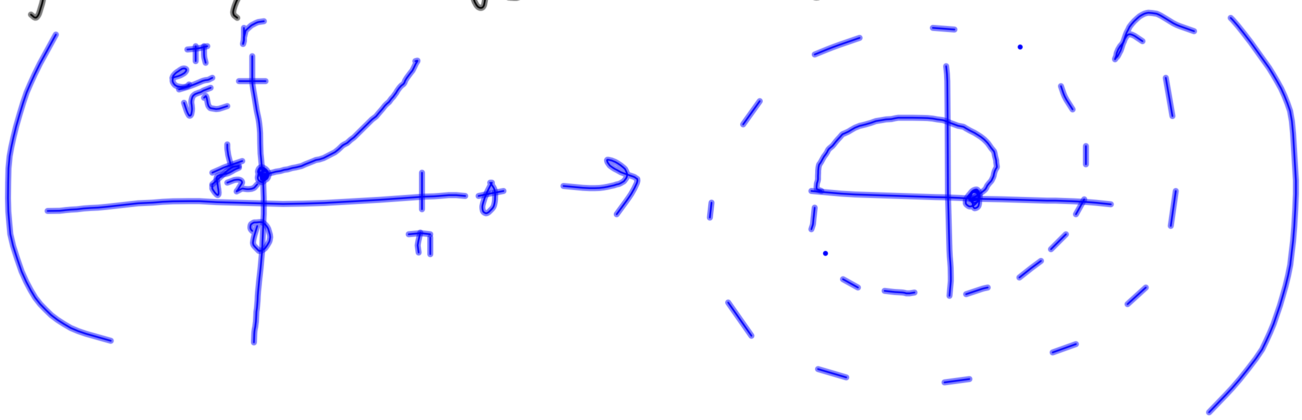
$$= \int_0^{2\pi} 2\sin\frac{\theta}{2}\, d\theta$$

$$= \left[-2\cos\frac{\theta}{2}(2)\right]_0^{2\pi} = \left[-4\cos\frac{\theta}{2}\right]_0^{2\pi}$$

$$= -4\cos\pi + 4\cos 0$$

$$= -4(-1) + 4(1) = \boxed{8}$$

Find the length of the polar graph given by $r = \frac{e^\theta}{\sqrt{2}}$ for $0 \le \theta \le \pi$.



$$L = \int_0^\pi \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\, d\theta \qquad \frac{dr}{d\theta} = \frac{e^\theta}{\sqrt{2}}$$

$$= \int_0^\pi \sqrt{\left(\frac{e^\theta}{\sqrt{2}}\right)^2 + \left(\frac{e^\theta}{\sqrt{2}}\right)^2}\, d\theta$$

$$= \int_0^\pi \sqrt{\frac{e^{2\theta}}{2} + \frac{e^{2\theta}}{2}}\, d\theta$$

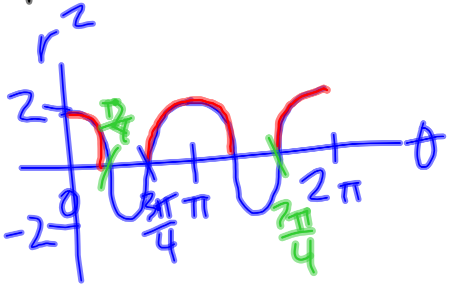
$$= \int_0^\pi \sqrt{e^{2\theta}}\, d\theta$$

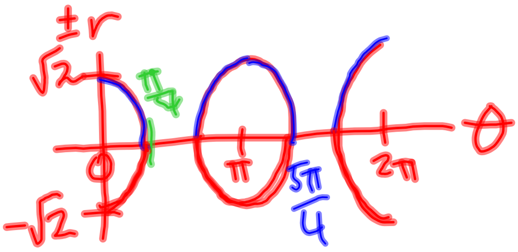
$$= \int_0^\pi (e^{2\theta})^{1/2}\, d\theta = \int_0^\pi e^\theta\, d\theta$$

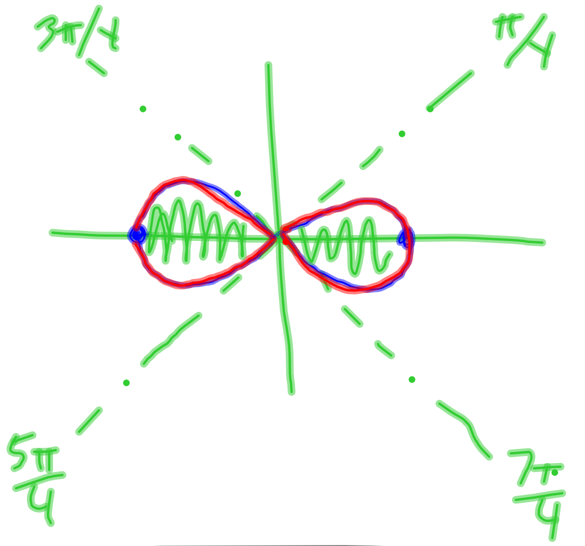
$$= \left[e^\theta\right]_0^\pi = e^\pi - e^0 = \boxed{e^\pi - 1}$$

Find the area inside the graph of

$r^2 = 2\cos 2\theta$. $\left(-\frac{\pi}{4} \text{ to } \frac{\pi}{4} \text{ and } \frac{3\pi}{4} \text{ to } \frac{5\pi}{4}\right)$







$$A = 2\int_{-\pi/4}^{\pi/4} \tfrac{1}{2} r^2 \, d\theta$$

$$= \cancel{2}\int_{-\pi/4}^{\pi/4} \cancel{\tfrac{1}{2}}(2\cos 2\theta)\, d\theta$$

$$= \int_{-\pi/4}^{\pi/4} 2\cos 2\theta \, d\theta$$

$$= \Big[\sin 2\theta\Big]_{-\pi/4}^{\pi/4}$$

$$= \sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right) = 1 - -1 = \boxed{2}$$