7.2: Trigonometric Integrals

Idea: Combining the use of trigonometric identites with substitution and integration by parts to solve complicated trig integrals.

First strategy: using Pythagorean Identies

$$a^2 + b^2 = c^2$$

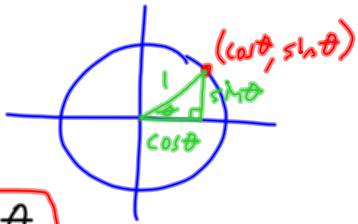
$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$\downarrow \div$ by $\cos^2$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

$\div$ by $\sin^2$

$$\boxed{1 + \cot^2\theta = \csc^2\theta}$$



Useful for finding:
when one power is odd.

$$\int \sin^{m}(x)\cos^{n}(x)\,dx$$

Ex!

$$\int \sin^3(x)\cos^2(x)\,dx$$

Idea: Replace all sine functions with cosine functions except for a single sin(x)dx.

$$= \int \sin^2(x)\cos^2(x) \cdot \sin(x)\,dx$$

Now, use $\sin^2(x) = 1 - \cos^2(x)$

$$= \int \left(1 - \cos^2(x)\right)\cos^2(x) \cdot \sin(x)\,dx$$

Now I can use substitution.

Let $u = \cos(x)$

$du = -\sin(x)\,dx$

$-du = \sin(x)\,dx$

$$= \int (1 - u^2)u^2(-du)$$

$$= \int (u^2 - 1)u^2\,du$$

$$= \int u^4 - u^2\,du$$

$$= \tfrac{1}{5}u^5 - \tfrac{1}{3}u^3 + C$$

$$= \boxed{\tfrac{1}{5}\cos^5(x) - \tfrac{1}{3}\cos^3(x) + C}$$

$$\int \tan^4 x \, dx$$

$$= \int \tan^2 x \tan^2 x \, dx$$

$$= \int \tan^2 x (\sec^2 x - 1) dx$$

$$= \int \tan^2 x \sec^2 x - \tan^2 x \, dx$$

$$= \int \tan^2 x \sec^2 x \, dx - \int \tan^2 x \, dx$$

$$= \int \tan^2 x \sec^2 x \, dx - \int \sec^2 x - 1 \, dx$$

Let $u = \tan x$

$du = \sec^2 x \, dx$

$$= \int u^2 du - [\tan x - x]$$

$$= \frac{1}{3} u^3 - \tan x + x + C$$

$$= \frac{1}{3} \tan^3 x - \tan x + x + C$$