Integration by Parts Examples

$$\int_0^{\pi} y^2 \sin(2y)\,dy \qquad \text{Let } u = y^2 \quad v' = \sin(2y)$$
$$u' = 2y \quad v = -\tfrac{1}{2}\cos(2y)$$

$$\boxed{\int u \cdot v'\,dx = uv - \int u' \cdot v\,dx}$$

$$= \left[y^2\left(-\tfrac{1}{2}\cos(2y)\right) - \int \cancel{2}y\left(-\tfrac{1}{\cancel{2}}\cos 2y\right)dy\right]_0^{\pi}$$

$$= \left[-\tfrac{1}{2}y^2\cos(2y) + \int y\cos(2y)\,dy\right]_0^{\pi}$$

$$\text{Let } u = y \quad v' = \cos(2y)$$
$$u' = 1 \quad v = \tfrac{1}{2}\sin(2y)$$

$$= \left[-\tfrac{1}{2}y^2\cos(2y) + \left[\tfrac{1}{2}y\sin(2y) - \int \tfrac{1}{2}\sin(2y)\,dy\right]\right]_0^{\pi}$$

$$= \left[-\tfrac{1}{2}y^2\cos(2y) + \tfrac{1}{2}y\sin(2y) - \tfrac{1}{2}\left(-\tfrac{1}{2}\cos 2y\right)\right]_0^{\pi}$$

$$= \left(-\tfrac{1}{2}\pi^2\cancel{\cos(2\pi)} + \cancel{\tfrac{1}{2}\pi\sin(2\pi)} + \tfrac{1}{4}\cancel{\cos(2\pi)}\right)$$

$$- \left(0 + 0 + \tfrac{1}{4}\cancel{\cos(0)}\right)$$

$$= \left(-\tfrac{1}{2}\pi^2 + \cancel{\tfrac{1}{4}}\right) - \cancel{\tfrac{1}{4}} = \boxed{-\tfrac{1}{2}\pi^2}$$

$$\int e^{2x}\cos(3x)\,dx$$

Let $u=\cos(3x)$ $\quad v'=e^{2x}$

$u'=-3\sin(3x)$ $\quad v=\frac{1}{2}e^{2x}$

$= \cos(3x)\left(\frac{1}{2}e^{2x}\right)+\int 3\sin(3x)\frac{1}{2}e^{2x}\,dx$

$= \frac{1}{2}\cos(3x)e^{2x}+\int \frac{3}{2}\sin(3x)e^{2x}\,dx$

Let $u=\frac{3}{2}\sin(3x)$ $\quad v'=e^{2x}$

$u'=\frac{9}{2}\cos(3x)$ $\quad v=\frac{1}{2}e^{2x}$

$= \frac{1}{2}\cos(3x)e^{2x}+\frac{3}{4}\sin(3x)e^{2x}-\int \frac{9}{4}\cos(3x)e^{2x}\,dx$

$$\int e^{2x}\cos(3x)\,dx = \frac{1}{2}\cos(3x)e^{2x}+\frac{3}{4}\sin(3x)e^{2x}-\frac{9}{4}\int \cos(3x)e^{2x}\,dx$$

$$\int e^{2x}\cos(3x)\,dx+\frac{9}{4}\int \cos(3x)e^{2x} = \frac{1}{2}\cos(3x)e^{2x}+\frac{3}{4}\sin(3x)e^{2x}$$

$$\frac{13}{4}\underbrace{\int e^{2x}\cos(3x)\,dx} = \frac{1}{2}\cos(3x)e^{2x}+\frac{3}{4}\sin(3x)e^{2x}+C$$

$$\int e^{2x}\cos(3x)\,dx = \frac{4}{13}\left[\frac{1}{2}\cos(3x)e^{2x}+\frac{3}{4}\sin(3x)e^{2x}\right]+C$$

$$= \frac{2\cos(3x)e^{2x}+3\sin(3x)e^{2x}}{13}+C$$

$$\int \sin(\ln x)\,dx$$

$$\int \frac{e^{\ln x}\sin(\ln x)}{x}\,dx = \int e^{u}\sin(u)\,du$$

Let $u = \ln x$

$du = \frac{1}{x}dx$

Solve using int by parts

$$= \frac{\cos(u)e^{u} - \sin(u)e^{u}}{2} + C$$

Not sure

$$= \frac{\cos(\ln x)\cancel{e}^{\cancel{\ln} x} - \sin(\ln x)\cancel{e}^{\cancel{\ln} x}}{2} + C$$

$$\boxed{= \frac{x\cos(\ln x) - x\sin(\ln x)}{2} + C}$$