Integral Test (8.3)

Idea: There's a relationship between a series of **nonnegative** terms and a **nondecreasing** sequence.

Suppose we have a series of nonnegative terms.

$$\sum_{n=1}^{\infty} a_n \,, \quad a_i \geq 0 \text{ for all integers } i$$

I remember that a series is defined to be the limit of the partial sum sequence.

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n$$

$$s_{n+1} = \sum_{i=1}^{n+1} a_i = \left(\sum_{i=1}^{n} a_i\right) + a_{n+1}$$

nonnegative

$$= s_n + a_{n+1} \geq s_n$$

So $s_n \leq s_{n+1}$, and s_n is a nondecreasing sequence.

Conclusion:

If a_n is a sequence of **nonnegative** terms, then $s_n = \sum_{i=1}^{n} a_i$ is a **nondecreasing** sequence.

Thus $\sum_{i=1}^{\infty} a_i$ converges if and only if it has an upper bound.

Harmonic Series:

$$\sum_{n=1}^{\infty} \frac{1}{n}$$ is the Harmonic Series.

$$= \frac{1}{1} + \frac{1}{2} + \underbrace{\frac{1}{3} + \frac{1}{4}} + \underbrace{\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}} + \cdots$$

$$\geq \frac{1}{1} + \frac{1}{2} + \underbrace{\frac{1}{4} + \frac{1}{4}} + \underbrace{\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}} + \cdots$$

$$1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots$$

$$\downarrow$$

$$\infty$$

So the harmonic series diverges.