Actual Example of using the Integral Test (8.3)

$$\sum_{i=1}^{\infty} \frac{1}{i(1+(\ln i)^2)} \leftarrow \text{positive } \& \text{ decreasing}$$

By the integral test, this converges/diverges if and only if the similar integral converges/diverges.

$$\int_1^{\infty} \frac{1}{x(1+(\ln x)^2)}\,dx = \lim_{b\to\infty} \int_1^b \frac{1}{x(1+(\ln x)^2)}\,dx$$

Let $u = \ln x$
$du = \frac{1}{x}dx$

$$= \lim_{b\to\infty} \int_{x=1}^{x=b} \frac{du}{1+u^2}$$

$$= \lim_{b\to\infty} \Big[\text{Arctan}\, u\Big]_{x=1}^{x=b}$$

$$= \lim_{b\to\infty} \Big[\text{Arctan}(\ln x)\Big]_1^b$$

$$= \lim_{b\to\infty} \Big[\text{Arctan}(\ln b) - \text{Arctan}(\ln 1)\Big]$$

$$= \lim_{b\to\infty} \Big[\text{Arctan}(\ln b) - \cancel{\text{Arctan}(0)}\Big]$$

$$= \lim_{b\to\infty} \text{Arctan}(\ln b)$$

$$= \text{Arctan}(\ln \infty)$$

$$= \text{Arctan}(\infty)$$

$$= \pi/2$$

So $\displaystyle\int_1^{\infty} \frac{1}{x(1+(\ln x)^2)}\,dx$ converges

$$\Downarrow$$

Thus $\displaystyle\sum_{i=1}^{\infty} \frac{1}{i(1+(\ln i)^2)}$ also converges.