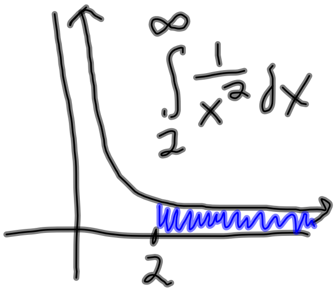
Improper Integrals (7.7) continued

Type I Improper Integrals dealt with an infinite limit of integration



$$\int_2^\infty \frac{1}{x^2}\,dx = \lim_{b\to\infty}\int_2^b \frac{1}{x^2}\,dx$$

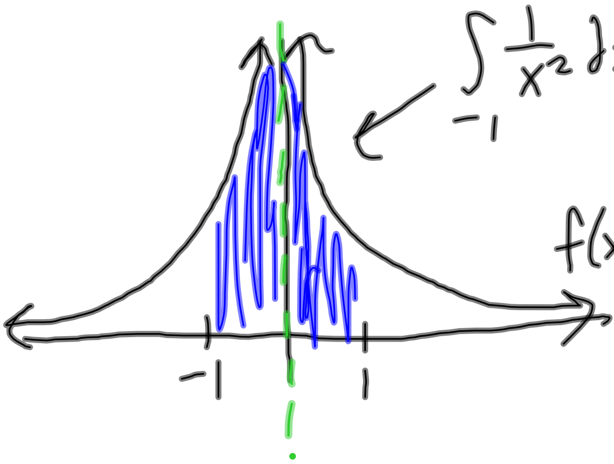
converges if $p>1$

$$\int_a^\infty \frac{1}{x^p}\,dx$$ diverges if $p \le 1$

Type II Improper Integrals will deal with vertical asymptotes.

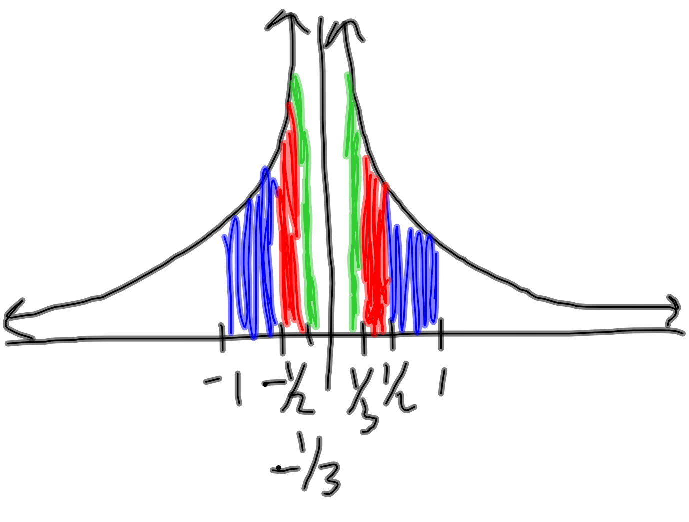
$$\int_{-1}^{1} \frac{1}{x^2}\,dx \neq \left[-\frac{1}{x}\right]_{-1}^{1}$$

Because $-1 \le 0 \le 1$ is undefined



$f(x) = 1/x^2$

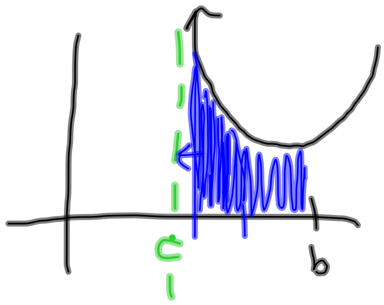
Idea: we're going to "estimate" the area by taking limits of the area as we approach the asymptote



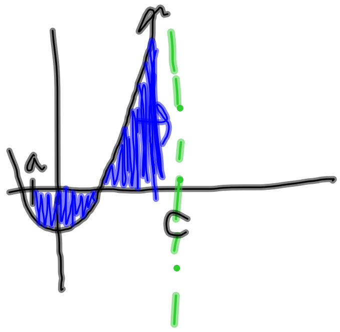
Improper Integrals (Type II)

If there is an asymptote at x=c...

$$\int_c^b f(x)\,dx = \lim_{a\to c^+}\int_a^b f(x)\,dx$$

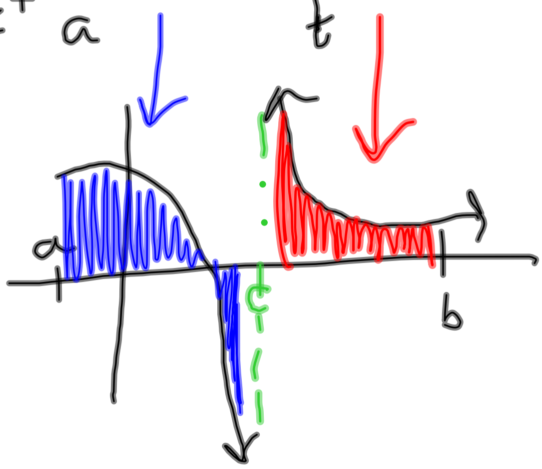


$$\int_a^c f(x)\,dx = \lim_{b\to c^-}\int_a^b f(x)\,dx$$

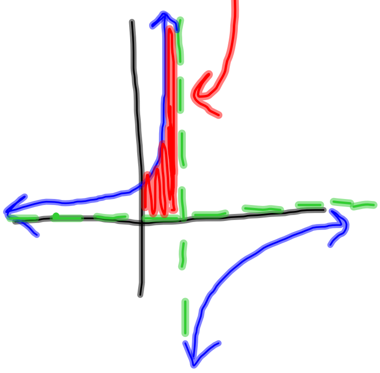


If $a \le c \le b$

$$\int_a^b f(x)\,dx = \lim_{\substack{s\to c^- \\ t\to c^+}} \int_a^s f(x)\,dx + \int_t^b f(x)\,dx$$



$$\int_0^1 \frac{1}{1-x}\,dx$$

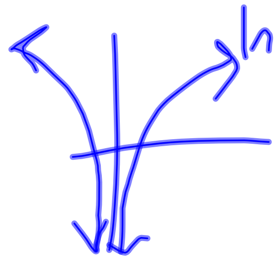


Since there's an asymptote at x=1, we have to use the Type II integral defintion

$$= \lim_{b\to 1^-} \int_0^b \frac{1}{1-x}\,dx$$

$$= \lim_{b\to 1^-} \Big[-\ln|1-x|\Big]_0^b$$

$$= \lim_{b\to 1^-} \Big[-\ln|1-b| + \cancel{\ln|1+0|}\Big]$$



$$= \lim_{b\to 1^-} -\ln|1-b|$$

$$= +\left(+\boxed{\infty}\right)$$ diverges

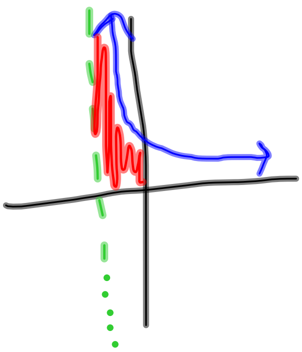
As $b\to 1^-$ what happens to $1-b$? → 0^+

So $\lim_{b\to 1^-} -\ln|1-b| = \lim_{1-b\to 0^+} -\ln|1-b|$

$$\int_{-1}^{0} \frac{1}{(x+1)^{3/2}} = \lim_{a\to -1^+} \int_{x=a}^{x=0} \frac{1}{(x+1)^{3/2}}\,dx$$

Let $u = x+1$ $x=0 \to u=1$

$du = dx$ $x=a \to u=a+1$



$$= \lim_{a\to -1^+} \int_{u=a+1}^{u=1} \frac{1}{u^{3/2}}\,du$$

$$= \lim_{a\to -1^+} \left[-2u^{-1/2}\right]_{a+1}^{1}$$

$$= \lim_{a\to -1^+} \left[-2 + \left(+\frac{2}{\sqrt{a+1}}\right)\right]$$

Diverge

Going to ∞ $\Leftarrow$ Denominator of 0

$$\int_{-1}^{0} \frac{1}{(x+1)^{1/2}}\,dx = \lim_{a\to -1^+} \int_{a}^{0} \frac{1}{(x+1)^{1/2}}\,dx$$

$$= \lim_{a\to 1^+} \int_{a+1}^{1} u^{-1/2}\,dx$$

$$= \lim_{a\to 1^+} \left[2u^{1/2} \right]_{a+1}^{1}$$

$$= \lim_{a\to 1^+} \left[2 - 2\sqrt{a+1} \right]$$

$$\downarrow \qquad \downarrow$$

$$2 - 0$$

$$\boxed{2}$$

$$\int_0^1 \frac{1}{x^p}\,dx \quad \begin{array}{l}\text{converges for } p<1\\ \text{diverges for } p\geq 1\end{array}$$

Proof of this is similar to what I did last time.