Clarification:

If $\sum_{i=N}^{\infty} a_i$ converges (diverges) for a particular integer N,

then $\sum_{i=N}^{\infty} a_i$ converges (diverges) for all integers N.

Example:

If I proved $\sum_{n=5}^{\infty} \frac{\ln n}{n+1}$ diverges,

then I also showed $\sum_{n=1}^{\infty} \frac{\ln n}{n+1}$ diverges.

8.4 Comparison Tests for Series

Idea: Because series converge/diverge similarly to similar-looking integrals, it stands to reason that the convergence test for integrals (Direct Comparison & Limit Comparison) also apply to series.

Direct Comparison Test for Series

Let $\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$ be series with nonnegative terms such that $a_i \leq b_i$ for all integers i.

- If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges
- If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

Ex: Does this converge/diverge?

$$\sum_{k=2}^{\infty} \frac{5}{5k-1}$$

Direct Comparison Test for Series

Let $\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$ be series with nonnegative terms such that $a_i \le b_i$ for all integers i.

- If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges
- If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

- All terms are nonnegative.
- I know $\frac{5}{5k-1} \ge \frac{5}{5k}$ ← (R denom. is bigger than L denom.)
- I know $\sum_{k=2}^{\infty} \frac{5}{5k} = \sum_{k=2}^{\infty} \frac{1}{k}$ diverges.
- Thus the bigger $\sum_{k=2}^{\infty} \frac{5}{5k-1}$ also diverges.

$$\sum_{n=0}^{\infty} \frac{1}{n!}$$

Direct Comparison Test for Series

Let $\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$ be series with nonnegative terms such that $a_i \le b_i$ for all integers i.

- If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges
- If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

- All terms are nonnegative.
- $\frac{1}{n!} \le \frac{1}{2^{n-1}}$
- $\sum_{n=1}^{\infty} \frac{1}{2^{n-1}}$ is a geometric series with $r = 1/2 < 1$ so it converges.
- Thus the smaller $\sum_{n=0}^{\infty} \frac{1}{n!}$ also converges.

Limit Comparison Test for Series

Let $\sum_{n=0}^{\infty} a_n$, $\sum_{n=0}^{\infty} b_n$ be series w/ nonnegative terms.

- If $0 < \lim_{n\to\infty} \frac{a_n}{b_n} < \infty$, then $\sum_{n=0}^{\infty} a_n$, $\sum_{n=0}^{\infty} b_n$ both converge or both diverge.

- If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum_{n=0}^{\infty} b_n$ converges, then $\sum_{n=0}^{\infty} a_n$ converges.

- If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum_{n=0}^{\infty} b_n$ diverges, then $\sum_{n=0}^{\infty} a_n$ diverges.