Cal II Extra Credit 1 (Spring 2010, Instructor: Steven Clontz)

In class, I defined an improper integral of type II to be this:

(a) $\int_a^b f(x)dx = \lim_{s \to c^- \text{ and } t \to c^+} \int_a^s f(x)dx + \int_t^b f(x)dx$ (given some vertical asymptote at c between a and b)

Strictly speaking, you haven't been told how to handle limits with two limiting variables, so perhaps this isn't well-defined. Instead, I could have defined it as

(b) $\int_a^b f(x)dx = \lim_{s \to c^-} \int_a^s f(x)dx + \lim_{t \to c^+} \int_t^b f(x)dx$

or

(c) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

The issue about these is that we're always dealing with two limiting variables, and I don't really have nice rules (like L'Hopital) to handle limits like

$$\lim_{s \to 0^- \text{ and } t \to 0^+} \frac{s}{t}$$

which could easily pop up. So, I claim that the following is also a valid definition of a Type II improper integral with a VA at c:

(d) $\int_a^b f(x)dx = \lim_{s \to c^-} \int_a^s f(x)dx + \int_{2c-s}^b f(x)dx$

If true, this would make evaluating the limits more straightforward, since I can use L'Hopital.

1. **For 1/2 extra point added to your course grade:**
   Show why (a),(b), and (c) all mean the same thing.
2. **For 1 extra point added to your course grade:**
   Show why (a),(b),(c), and (d) all mean the same thing.